

# The $\mathbb{Z}_2$ Two Higgs Doublet Model and the hierarchy problem

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**ABSTRACT:** The hierarchy problem is investigated in the  $\mathbb{Z}_2$  Two Higgs Doublet extension of the Standard Model. From the quadratic divergences of the Higgs field propagators fine-tuning constraints are extracted and solved. This leads to the mass relation  $m_{h^0}^2 < 2m_t^2 < m_{H^0}^2$ , where  $h^0$  is the analogue of the SM Higgs boson and  $H^0$  a heavier neutral scalar. The constraint leads to an admissible condition on the Higgs boson mass, in contrast to a similar treatment for the Standard Model. Based on this analysis it is argued that there may not be a little hierarchy problem for the  $\mathbb{Z}_2$  model, as it is the low energy counterpart of the MSSM.

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## 1 Introduction

The mechanism for the introduction of masses for the Standard Model fermions and gauge bosons remains experimentally unverified. Although the Higgs mechanism is the most promising approach, there is still a lot of freedom to extend the Standard Model. However, the Higgs mechanism suffers from the hierarchy problem, i.e. the absence of hierarchy in energy scales of physical theories. At energies of 100 GeV physics is described by the experimentally confirmed Standard Model, though beyond these energies only the GUT scale at  $10^{15}$  GeV — where the electroweak and strong couplings meet — and the Planck scale at  $10^{19}$  GeV — where quantum gravity appears — are conjectured. In summary, beyond the Standard Model there is an unnatural “energy desert” covering over twelve orders of magnitude.

## 2 Hierarchy and fine-tuning

In the Standard Model the lack of hierarchy translates to a problem called fine-tuning. The Higgs boson mass receives quadratically divergent corrections and the one-loop correction equals [1]

$$\delta m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left( \frac{3m_h^2}{16} + \frac{3m_W^2}{8} + \frac{3m_Z^2}{16} - \frac{3m_t^2}{4} \right), \quad (2.1)$$

where  $v \approx 246$  GeV and  $\Lambda$  is the momentum cut-off in the loop. The physical mass of the Higgs boson, equal to the pole of the propagator, is given by

$$m_{h,\text{phys}}^2 = m_h^2 + \delta m_h^2. \quad (2.2)$$

Electroweak theory predicts a physical mass around 100 GeV, which receives a contribution of approximately  $10^{15}$  GeV, as the GUT scale is inserted for  $\Lambda$ . For the physical mass to stay around the electroweak scale the one-loop contribution must be cancelled by the bare parameter in the Lagrangian. This implies a fine-tuning of 28 decimal places in this bare parameter. In conclusion, the “energy desert” manifests itself as a fine-tuning problem in the Higgs sector of the Standard Model. Many attempts to solve this issue have been made, two of them are reviewed in the following sections.

## 2.1 Fine-tuning constraints

The first approach is obtained by demanding the absence of quadratic divergences in a theory, which leads to constraints on that theory [2, 3]. For the Standard Model there is one constraint, which reads:

$$m_h^2 = 4m_t^2 - 2m_W^2 - m_Z^2. \quad (2.3)$$

With the fermion and gauge boson masses known from experiments [4], the mass of the Higgs boson is required to equal  $m_h = 311.6 \pm 3.5$  GeV. This value is unrealistic within the known mass bounds

$$114 \text{ GeV} < m_h < 185 \text{ GeV}, \quad (2.4)$$

obtained by the LEP Electroweak Working Group [5]. Note that the upper bound from perturbation theory only holds in the framework of the Standard Model.

**Exact solution** The fine-tuning constraints originate from one-loop corrections, for an exact solution of the fine-tuning problem the discussion should be extended to any number of loops. In [6] it is argued that two scenarios may appear: the cancellation of quadratic divergences order by order and cancellation up to a certain order. The first scenario implies an infinite number of constraints that must be satisfied. At first sight the second scenario seems to differ, however, it is shown that also here an infinite number of constraints appear [6]. Satisfying an infinite number of constraints is highly unfeasible unless there is a symmetry in the theory, e.g. supersymmetry. In conclusion, an exact solution to the fine-tuning problem is not obtainable through the method of adopting fine-tuning constraints.

**Alleviating fine-tuning** Instead of attaining an exact solution, the fine-tuning problem may be lessened. Through perturbation theory it is argued in [7] that one-loop corrections dominate the fine-tuning problem up to a certain energy scale  $\Lambda$ . Then, if the one-loop corrections vanish, i.e. the fine-tuning constraints are satisfied, fine-tuning is absent below that energy scale. Beyond that energy scale the amount of fine-tuning is reduced, and the problem is alleviated.

Actually, it is enlightening to determine the energy scale below which no fine-tuning is necessary. That energy scale is denoted as  $\Lambda^{\text{NP}}$ , the scale at which new physics should appear for the fine-tuning problem to be nonexistent. In the Standard Model no fine-tuning is present if the one-loop correction to the Higgs boson mass (2.1) is roughly less than the electroweak scale  $v/2$ , hence it is required that

$$\left| \frac{1}{16\pi^2 v^2} \left( \Lambda_{\text{SM}}^{\text{NP}} \right)^2 \left( 3m_h^2 + 6m_W^2 + 3m_Z^2 - 12m_t^2 \right) \right| \lesssim \frac{v^2}{4}. \quad (2.5)$$

Insertion of the experimental Higgs mass bounds (2.4) gives an energy scale of new physics  $\Lambda_{\text{SM}}^{\text{NP}} \approx 750 - 900 \text{ GeV}$ .

## 2.2 Minimal Supersymmetric Standard Model

The straightforward approach to the fine-tuning problem is removing all quadratic divergences in a theory. Supersymmetry meets this demand through the introduction of a symmetry between bosons and fermions. The simplest supersymmetric extension of the Standard Model is the Minimal Supersymmetric Standard Model (MSSM). For each of the particles in the Standard Model a superpartner is introduced in order to cancel its quadratic divergent corrections. Due to the nature of supersymmetry two scalar doublets are needed to provide masses for both types of quarks, up and down. The problem of fine-tuning is absent in the MSSM, although it still suffers from another problem: the little hierarchy problem.

**Little hierarchy problem** The little hierarchy problem occurs in the transition region from an effective low energy theory (SM) and its supersymmetric counterpart (MSSM), for a review see [8]. Supersymmetry must be broken at some scale  $\Lambda_{\text{SUSY}}$ , since superpartners have not been found in experiments yet. Above this energy scale supersymmetry is exact, quadratic divergences are cancelled and there is no need for fine-tuning. The same holds at Standard Model energies  $\Lambda_{\text{SM}}$ , since the loop corrections and the physical Higgs mass are of the same order. However, a problem appears just below the scale of supersymmetry breaking where loop corrections are of order  $\Lambda_{\text{SUSY}}$ . This implies a fine-tuning of roughly  $\Lambda_{\text{SUSY}}^2/\Lambda_{\text{SM}}^2$ .

An estimate of the supersymmetry breaking scale can be obtained from the upper bound on the mass of the lightest scalar  $h^0$  in the MSSM. The bound  $m_{h^0} < m_Z$  is inconsistent with the lower bound on the Higgs mass (2.4), however loop corrections from sparticles can raise the theoretical upper bound on  $m_{h^0}$  [8]. To raise the upper bound to a realistic value above the LEP bound, sparticle masses and thereby the supersymmetry breaking scale must be around 1.5 TeV. Since  $\Lambda_{\text{SUSY}} > \Lambda_{\text{SM}}^{\text{NP}}$ , the transition from Standard Model to MSSM does not occur without a little fine-tuning.

## 3 $\mathbb{Z}_2$ Two Higgs Doublet Model

Two Higgs Doublet Models (2HDMs) have two left-handed scalar doublets and were first introduced in [9]. These models appear in different types, for comprehensive overviews see [10, 11]. To analyse the hierarchy problem and related topics as discussed in section 2 a specific model needs to be chosen. The  $\mathbb{Z}_2$  Two Higgs Doublet Model is chosen, the justification for this choice is based on the facts that it:

- is the minimal non-trivial extension to the Standard Model Higgs sector.
- resembles the Standard Model Higgs sector as CP is conserved and tree-level FCNCs are absent, beyond tree-level FCNCs are suppressed by the GIM mechanism [12].

- equals the MSSM Higgs sector if the Yukawa Lagrangian is type-II and if the potential satisfies additional constraints compared to the  $\mathbb{Z}_2$  model discussed below [13].

A brief overview of the  $\mathbb{Z}_2$  Two Higgs Doublet Model will be given to clarify some aspects relevant for the discussion of the hierarchy problem.

### 3.1 2HDM Lagrangian

The Lagrangian of the  $\mathbb{Z}_2$  2HDM is the sum of the massless Standard Model Lagrangian and the Lagrangian defined as

$$\mathcal{L}_{2\text{HDM}} = (D_\mu \Phi_1)^\dagger D^\mu \Phi_1 + (D_\mu \Phi_2)^\dagger D^\mu \Phi_2 - V(\Phi_1, \Phi_2) + L_Y \quad (3.1)$$

The Lagrangian should be invariant under both CP and a  $\mathbb{Z}_2$  symmetry, which is introduced between the doublets as  $\Phi_1 \rightarrow \Phi_1$ ,  $\Phi_2 \rightarrow -\Phi_2$ . The kinetic terms are invariant under these symmetries, however, for the potential this implies a reduction from fourteen to seven parameters [10]. With the four gauge invariant combinations of the scalar doublets defined as

$$A \equiv \Phi_1^\dagger \Phi_1, \quad B \equiv \Phi_2^\dagger \Phi_2, \quad C \equiv \text{Re}(\Phi_1^\dagger \Phi_2), \quad D \equiv \text{Im}(\Phi_1^\dagger \Phi_2), \quad (3.2)$$

the potential is given by

$$V(\Phi_1, \Phi_2) = -\mu_1^2 A - \mu_2^2 B + \lambda_1 A^2 + \lambda_2 B^2 + \lambda_3 C^2 + \lambda_4 D^2 + \lambda_5 AB. \quad (3.3)$$

In the most general Yukawa Lagrangian both doublets couple to all fermion types — up-type quarks, down-type quarks and charged leptons. The  $\mathbb{Z}_2$  symmetry restricts these couplings such that only one doublet may couple to one type of fermion, dependent on how the symmetry is introduced for the fermions. To resemble the MSSM, Type-II Yukawa interactions are employed. In the fermionic sector the  $\mathbb{Z}_2$  symmetry has to be introduced as  $e_R, d_R \rightarrow e_R, d_R$  and  $u_R \rightarrow -u_R$ , which leads to a Yukawa Lagrangian of the form

$$\mathcal{L}_Y = -\lambda_e \bar{l}_L \Phi_1 e_R - \lambda_d \bar{q}_L \Phi_1 d_R - \lambda_u \bar{q}_L i \sigma_2 \Phi_2^* u_R + \text{h.c.} \quad (3.4)$$

The  $\mathbb{Z}_2$  symmetry ensures absence of Flavour Changing Neutral Currents in the Yukawa sector [14] and CP violation in the potential [15]. The only source for CP violation in this model resides, as in the Standard Model, in the complex phase of the CKM matrix.

### 3.2 2HDM Phenomenology

In order to obtain the mass states it is necessary to find the minima of the potential, both doublets then acquire a vacuum expectation value just as in the Standard Model. There are two possibilities

$$v_1 = \frac{\mu_1^2}{\lambda_1}, \quad v_2 = 0 \quad \text{and} \quad v_1^2 = \frac{\lambda_2 \mu_1^2 - \lambda_+ \mu_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \quad v_2^2 = \frac{\lambda_1 \mu_2^2 - \lambda_+ \mu_1^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \quad (3.5)$$

where  $\lambda_+ \equiv \frac{1}{2}(\lambda_3 + \lambda_5)$ . The choice where either  $v_1$  or  $v_2$  is zero leads to the Inert 2HDM (IDM) [16]. After spontaneous symmetry breaking in the potential there are three massless

Goldstone scalars  $G^\pm, G^0$  corresponding to the weak gauge bosons and five massive scalars  $H^\pm, H^0, h^0, A^0$ . The masses of the massive scalars in terms of the potential parameters read

$$\begin{aligned}
m_{H^\pm}^2 &= -\frac{1}{2}\lambda_3(v_1^2 + v_2^2) \\
m_{H^0}^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4\lambda_+^2 v_1^2 v_2^2} \\
m_{h^0}^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 - \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4\lambda_+^2 v_1^2 v_2^2} \\
m_{A^0}^2 &= \frac{1}{2}(\lambda_4 - \lambda_3)(v_1^2 + v_2^2).
\end{aligned} \tag{3.6}$$

The mass states originate from mixing between the doublets, which leads to two mixing angles  $\alpha, \beta$  defined as

$$\tan \beta \equiv \frac{v_2}{v_1}, \quad \tan 2\alpha \equiv \frac{2v_1 v_2 \lambda_+}{\lambda_1 v_1^2 - \lambda_2 v_2^2}. \tag{3.7}$$

There are two theoretical issues to address. The minimum of the potential must be stable, which requires all scalar masses to be positive. Furthermore the potential must be bounded from below, resulting in [17]

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_5 > -2\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_5 > -2\sqrt{\lambda_1 \lambda_2}, \quad \lambda_4 + \lambda_5 > -2\sqrt{\lambda_1 \lambda_2}. \tag{3.8}$$

As the mass states are known — the electroweak gauge mass states remain as in the Standard Model — the Lagrangian (3.1) can be expanded in terms of these mass states, from which the Feynman rules involving Higgs scalars can be straightforwardly obtained. They depend on six free parameters — the scalar masses and mixing angles — and  $v = v_1 + v_2 \approx 246$  GeV and the fermion and gauge boson masses. The Feynman rules for the  $\mathbb{Z}_2$  2HDM have been presented in [18], these have been rederived to provide a thorough check.

### 3.3 Fine-tuning in the $\mathbb{Z}_2$ 2HDM

In this section fine-tuning will be discussed in the context of the  $\mathbb{Z}_2$  2HDM. The fine-tuning constraints can not be satisfied consistently for the IDM [2, 19] and hence only the regular model will be treated.

**Fine-tuning constraints** To obtain the fine-tuning constraints as discussed in section 2.1 for the  $\mathbb{Z}_2$  2HDM, all quadratically divergent corrections to each of the six propagators are needed. This leads to six constraints, though only two of those are independent and read

$$\begin{aligned}
24m_t^2 \cos^2 \beta &= 6m_{H^0}^2 (\cos 2\beta - \cos 2\alpha) + 6m_{h^0}^2 (\cos 2\beta + \cos 2\alpha) \\
24m_t^2 \cos^2 \beta &= \sin^2 2\beta \left( 4m_{H^\pm}^2 + 2m_{A^0}^2 + 6m_W^2 + 3m_Z^2 \right) \\
&\quad + m_{H^0}^2 (6 - 6 \cos 2\alpha \cos 2\beta + 4 \sin 2\alpha \sin 2\beta) \\
&\quad + m_{h^0}^2 (6 + 6 \cos 2\alpha \cos 2\beta - 4 \sin 2\alpha \sin 2\beta).
\end{aligned} \tag{3.9}$$

The equations (3.9), together with the constraints (3.8), admit a solution for  $\alpha$  and  $\beta$  provided that:

$$\begin{aligned} 0 \leq m_{H^+}^2 < \infty, \quad 0 \leq m_{A^0}^2 < \infty, \\ 0 < m_{h^0}^2 < 2m_t^2 < m_{H^0}^2 < \infty. \end{aligned} \quad (3.10)$$

If these relations for the scalar masses hold, a unique solution in the mixing angles  $\alpha$  and  $\beta$  can be found numerically. This solution will satisfy the constraint  $\sin 2\beta > 0$  and is invariant under shifts over  $\pi$ . Hence, solutions are in correspondence with the experimental bounds on the Higgs mass (2.4).

**Little hierarchy problem** There is reason to expect that fine-tuning can be alleviated significantly compared to the Standard Model, since after the fine-tuning constraints have been satisfied additional free parameters are available in the  $\mathbb{Z}_2$  2HDM. For this model the calculations are extended to six different propagators, each of them receiving quadratically divergent corrections, leading to six equations equivalent to (2.5). Numerical analysis reveals that as long as equations (3.10) are satisfied, the scale of new physics for the  $\mathbb{Z}_2$  2HDM can be raised to  $\Lambda_{2\text{HDM}}^{\text{NP}} \approx 2.5$  TeV if the mixing angles  $\alpha$  and  $\beta$  are chosen in the neighbourhood of the unique solution.

In the analysis of the little hierarchy problem in section 2.2 a supersymmetry breaking scale of  $\Lambda_{\text{SUSY}} \approx 1.5$  TeV has been obtained. It was argued that a little fine-tuning is needed in the transition from the Standard Model to the MSSM. However, this is a rather naive analysis, since in fact the  $\mathbb{Z}_2$  2HDM is the effective low energy theory of the MSSM. Therefore one should consider the scale of new physics for the  $\mathbb{Z}_2$  2HDM, which results in  $\Lambda_{\text{SUSY}} \approx \Lambda_{2\text{HDM}}^{\text{NP}}$ . Now there is no need for any fine-tuning and the little hierarchy problem is absent.

## 4 Conclusions

The hierarchy problem has been analysed in terms of fine-tuning constraints, which arise from demanding the absence of quadratic divergences in a theory at one-loop order. It has been shown that these cannot be satisfied within experimental bounds (2.4) on the Higgs mass for the Standard Model. However, for the  $\mathbb{Z}_2$  Two Higgs Doublet Model a solution can be found provided that  $m_{h^0}^2 < m_t^2 < m_{H^0}^2$ .

It has been shown that the method of adopting the fine-tuning constraints is not able to solve the hierarchy problem to all orders. Nonetheless with this method the problem of fine-tuning may be alleviated, and accordingly a scale of new physics may be obtained below which there is no need for fine-tuning. For the Standard Model that energy scale is around 800 GeV. In the case of the  $\mathbb{Z}_2$  2HDM the situation is improved and the scale of new physics is around 2.5 TeV as long as  $m_{h^0}^2 < m_t^2 < m_{H^0}^2$  and the mixing angles are chosen suitably.

Furthermore it was argued that the little hierarchy problem may be absent, since the  $\mathbb{Z}_2$  2HDM is the low energy effective theory of the Minimal Supersymmetric Standard Model. For the  $\mathbb{Z}_2$  2HDM the scale of new physics can be lifted to the scale of supersymmetry breaking and there is no need for any fine-tuning.

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